FURYE QATORI. FURYE QATORI YORDAMIDA BA'ZI SONLI KETMA-KETLIKLARNING YIG'INDISINI TOPISH METODIKASI

Savronov Doston Rustam o'g'li, Ortiqboyev Dilmurod Zokir o'g'li
Guliston Shahridagi Prezident maktabi, Samarqand davlat universiteti

Annotatsiya. Ushbu maqola matematikaning eng muhim mavzulari bo'lgan Furye qatori va sonli ketma-ketliklar yig'indisiga doir qiziqarli muhim natijalarni o'z ichiga olgan. Dastlab Furye qatoriga yoyish tog'risida malumot keltirilgan va ba'zi sonli ketma-ketlilar yig'indisi topilgan. Agar $f(x)$ funksiya $[a;b]$ kesmada monoton bo'lsa yoki $[a;b]$ kesmani chekli sondagi qismiy kesmalarga bo'lish mumkin bo'lsa va bu kesmalarning har birida $f(x)$ funksiya monoton (faqat o'ssa yoki faqat kamaysa) yoki o'zgarmas bo'lsa, $f(x)$ funksiyaga $[a;b]$ kesmada bo'lakli monoton funksiya deyiladi. Agar $f(x)$ funksiya $[a;b]$ kesmada chekli sondagi birinchi tur uzilish nuqtalariga ega bo'lsa, $f(x)$ funksiyaga [a;b] kesmada bo'lakli-uzluksiz funksiya deyiladi. Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz yoki bo'lakli-uzluksiz bo'lib, bo'lakli-monoton bo'lsa $f(x)$ funksiya $[a;b]$ kesmada Dirixle shartlarini qanoatlantiradi deyiladi. Funksiyalarni Furye qatoriga yoyishda ular Dirixle shartlarini qanoatlantirishi zarur va yetarli.

Kalit so'zlar: Furye qatori, Furye koeffitsiyentlari. Funksiyalarni Furye qatoriga yoyish, sonli ketma-ketliklar va ularning yig'indisi.

Abstract. This paper contains interesting and important results on Fourier series and the sum of numerical sequences, which are the most important topics in mathematics. First, information on Fourier series propagation was given and the sum of some series of numbers was found. If the function $f(x)$ is monotonic in the section $[a;b]$ or if the section $[a;b]$ can be divided into a finite number of partial sections, and in each of these sections the function $f(x)$ is monotone (only increases or only decreases) or is constant, the function $f(x)$ is called a piecewise monotonic function on the cross section $[a;b]$. If the function $f(x)$ has a finite number of discontinuities of the first type on the section $[a;b]$, then the function $f(x)$ is called a piecewise-continuous function on the section $[a;b]$. If the function $f(x)$ is continuous or piecewise-continuous in the section $[a;b]$, and is piecewise-monotone, then the function $f(x)$ is said to satisfy the Dirichlet conditions in the section $[a;b]$. When expanding functions into a Fourier series, it is necessary and sufficient that they satisfy the Dirichlet conditions.

Key words: Fourier series, Fourier coefficients. Fourier expansion of functions, numerical sequences and their sum.

I.Fure qatorlari

Ortogonal ko'phadlar qatori matematikaning eng muhim sohalaridan biri bo'lib, ular turli differensial tenglamalar yechimlari ko'rinishida bo'lib, turli rekkurent munosabatlarga ega hamda funksiyalarni turli ko'rinishda ifodalashda foydali hamdir. Ulardan eng muhimlaridan biri Furye qatorlari bo'lib, funksiyani ortonormal triganometrik ko'phadlar ko'rinishida ifodalashdir. Agar $y = f(x)$ funksiya uzunligi 2π ga teng $(-\pi, \pi)$ oraliqda Dirixle shartlarini qanoatlantirsa, u holda bu oraliqning $f(x)$ uzluksiz bo'lgan har qanday x nuqtasida funksiyani Furye triganometrik qatoriga yoyish mumkin, ya'ni

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \qquad (1)$$

bu yerda a_n, b_n-Furye koeffitsentlari bo'lib, ular quyidagi formulalar bo'yicha olinadi

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx \quad (n = 0,1,2,\dots), \quad (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx \quad (n = 0,1,2,\dots), \quad (3)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx. \quad (4)$$

Agar $x_0 \in (-\pi, \pi)$ nuqta $f(x)$ funksiyaning uzilish nuqtasi bo'lsa, Furye qatori yig'indisi $S(x)$ funksiyaning chap va o'ng limitlari o'rta arifmetigiga teng bo'ladi

$$S(x) = \frac{1}{2}\left[\lim_{x\to x_0-0} f(x) + \lim_{x\to x_0+0} f(x)\right]$$

Oraliq chegaralarida

$$S(\pi) = S(-\pi) = \frac{1}{2}\left[\lim_{x\to -\pi+0} f(x) + \lim_{x\to \pi-0} f(x)\right] \quad (5)$$

Toq funksiyalar uchun $a_n = 0$ bo'lib Furye yig'indisi faqat sinuslarda, juft funksiyalar uchun esa $b_n = 0$ bo'lib Furye yig'indisi faqat kosinuslarda iborat bo'ladi.

II. Juft funksiyalarning Furye yoyilmasi yordamida ba'zi qatorlar yig'indisini hisoblash metodikasi

Toq funksiyalar Dirixle shartlarini qanoatlantirsa (5) ga muvofiq soha chegaralarida nolga teng bo'ladi. Shuning uchun biz juft funksiyalar qatorini qaraymiz va ba'zi juft funksiyalarni $(-\pi, \pi)$ oqaliqda Furye qatorlarini keltirib, unga mos qator yig'indisini soha chegaralarida hisoblaymiz.

1) $y = x^2$ funksiya. (2) formulaga ko'ra a_n ni topamiz va Furye qatorlarini hosil qilamiz

$$f(x) = x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$$

Ushbu tenglikning ikkala tomoniga $x = \pi$ yoki $x = -\pi$ almashtirish bajaramiz va $\cos nx = (-1)^n$ ekanligini hisobga olib quyidagi natijaga kelamiz

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

Haqiqatdan ham hisoblashlarning ko'rsatishicha $\pi^2/6 \approx 1.6449$ va $1/n^2$ qatorning dastlabki mingta hadining yig'indisi ≈ 1.64393 ga teng. Ushbu natija $\zeta(2)$ ning qiymatini ifodalaydi (ilovaga qarang).

2) $y = \cos \alpha x$ funksiya, bu yerda $\alpha \notin N$.

$$\cos \alpha x = \frac{\sin \alpha\pi}{\pi}\left[\frac{1}{\alpha} + \sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\alpha+n} + \frac{1}{\alpha-n}\right)\cos nx\right]$$

Yuqoridagi kabi almashtrishlardan keyin

$$\cos \alpha\pi = \frac{\sin \alpha\pi}{\pi}\left[\frac{1}{\alpha} + \sum_{n=1}^{\infty}\left(\frac{1}{\alpha+n} + \frac{1}{\alpha-n}\right)\right]$$

Demak

$$\sum_{n=1}^{\infty}\frac{1}{n^2-\alpha^2} = \frac{1}{2\alpha^2} - \frac{\pi}{2\alpha \tan \alpha\pi}$$

Ushbu ifoda yordamida α ning xussusiy natural bo‘lmagan qiymatlarida kerakli yig‘indini olishdan tashqari

$$\lim_{\alpha\to 0}\left(\frac{1}{2\alpha^2} - \frac{\pi}{2\alpha \tan \alpha\pi}\right) = \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

Qo‘shimcha natijaga ham ega bo‘lamiz

3) $y = x^4$ funksiya

$$x^4 = \frac{\pi^4}{5} + 8\sum_{n=1}^{\infty}(-1)^n\frac{\pi^2 n^2 - 6}{n^4}\cos nx$$

$x = \pi$ almashtirish bajaramiz va yuqoriagi $1/n^2$ hadlar yig‘indisidan foydalanamiz

$$\sum_{n=1}^{\infty}\left(\frac{\pi^2}{n^2} - \frac{6}{n^4}\right) = \frac{\pi^4}{10}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^2}{6}\sum_{n=1}^{\infty}\frac{1}{n^2} - \frac{\pi^4}{60} = \frac{\pi^4}{36} - \frac{\pi^4}{60} = \frac{\pi^4}{90}$$

Ushbu natija $\zeta(4)$ ning qiymatini ifodalaydi (ilovaga qarang).

4) $y = \cosh x$ funksiya

$$\cosh x = \frac{\sinh \pi}{\pi}\left(1 + 2\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2+1}\cos nx\right)$$

Yana $x = \pi$ almashtirishdan keyin

$$\sum_{n=1}^{\infty}\frac{1}{n^2+1} = \frac{1}{2}(\pi \coth \pi - 1)$$

5) $y = |x|$ funksiya

$$|x| = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}\cos nx$$

Bundan

$$\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}=2\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{4}$$

Yuqoridagi natijalardan foydalanib

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}=\sum_{n=1}^{\infty}\frac{1}{n^2}-\frac{\pi^2}{4}=\frac{\pi^2}{6}-\frac{\pi^2}{4}=-\frac{\pi^2}{12}$$

Ushbu tengliklar asosida quyidagi natijaga ham ega bo'lamiz

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$$

III. Juft bo'lmagan funksiyalar yordamida ba'zi qatorlar yig'indisini topish metodikasi

Toq bo'lmagan funsiyalarning ham $(-\pi,\pi)$ oraliqdagi Furye qatorlari ba'zi natijalarga olib kelishi mumkin

6) $f(x)=\begin{cases}-2, agar\ -\pi<x\le 0\ bo'lsa\\ 1, agar\ 0<x\le\pi\ bo'lsa\end{cases}$ funksiyani qaraymiz.

$$f(x)=-\frac{1}{2}-\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n}\sin nx=-\frac{1}{2}+\frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)x$$

Agar $x=\pi/2$ yoki $x=-\pi/2$ almashtirish bajarsak quyidagi Leybnits formulasiga (ilovaga qarang) kelamiz

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}=\frac{\pi}{4}$$

Eslatish joizki ushbu natijani biror toq funksiya yoyilmasidan ham hosil qilish mumkin (masalan $y=x$ funksiyaning $(-\pi,\pi)$ oraliqdagi yoyilmasi)

7) $y=x^3$ toq funksiyaning Furye yoyilmasini qaraymiz. Funksiya toq bo'lganligini uchun to'g'ridan-to'g'ri b_n ni (3) fo'rmuladan foydalanib hisoblaymiz

$$x^3=2\sum_{n=1}^{\infty}\left(\frac{6}{n^3}-\frac{\pi^2}{n}\right)(-1)^n\sin nx$$

$x=\pi/2$ almashtirishdan keyin quyidagi natijaga ega bo'lamiz

$$\sum_{n=1}^{\infty}\left(\frac{\pi^2}{2n-1}-\frac{6}{(2n-1)^3}\right)(-1)^{n+1}=\frac{\pi^3}{16}$$

va Leybnits formulasidan foydalanamiz

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^3}=\frac{\pi^2}{6}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}-\frac{\pi^3}{96}=\frac{\pi^3}{24}-\frac{\pi^3}{96}=\frac{\pi^3}{32}$$

VI. Xulosa

Yuqoridagi natijalar albatta avvaldan mavjud bo'lgan natijalarni takrori bo'lib ammo, ba'zi qatorlar yig'indisini Furye yoyilmasi yordamida osonroq topish mumkinligini ko'rsatadi. Boshqa biror funksiyani Furye qatoriga yoyish yana ushbu natijalarga olib kelishini ko'rsatadi. Ammo bu usulda qator yig'indisini topish boshqa usullarga qaraganda bir muncha osonroq hisoblanadi. Ushbu masalalar garmonik qator yig'indisini topishga urinishlardan biri sifatida ham qarash mumkin. Garmonik qator yig'indisi deb

$$S = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$$

yig'indiga aytiladi. Ko'plab fizik va matematik masalalar ushbu yig'indini topishga keltirilgani uchun taxminan XII asrlardan buyon ko'plab matematiklar tomonidan ushbu yig'indini topishga harakatlar bo'lgan. Bu yig'indining qismiy yig'indisi sifatida 6-masala natijasini qarash mumkin. Qismiy yig'indining chekli ekanligidan, yig'indining o'zi ham cheklidek ko'rnishi mumkin. Ammo ko'plab natijalar garmonik qator yig'indisi cheksiz ekanligini tasdiqlaydi. Masalan $\ln(1-x)$ funksiyaning $x_0 = 0$ nuqtadagi darajali qatorini olaylik

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$$

ushbu tenglikning ikkala tomonida $x = 1$ almashtirish

$$\sum_{n=1}^{\infty} \frac{1}{n} = -\ln 0 \to \infty$$

ekanligini ko'rsatadi.

V. Ilova

1. Riman zeta funksiyasi deb, haqiqiy qismi birdan katta bo'lgan s kompleks son uchun quyidagi yaqinlashuvchi va cheksiz ketma-ketlik yig'indisiga aytiladi

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

1730-yilda Leonard Eyler tomonidan $s > 1$ ning haqiqiy qiymatlari uchun yig'indini hisoblash formulasi topilgan. Bundan tashqari Eyler formulasi $s = 2n, n \in N$ hol uchun qulay modifikatsiya qilingan.

2. Yuqorida ta'kidlaganimizdek Leybnits formulasi garmonik qator yig'indisining qismiy yig'indisi hisoblanadi. Leybnits formulasi

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

ko'rinishda bo'lib birinchi marta π sonini hisoblash uchun 1673-yilda Jeyms Grigoriy va Leybnits tomonidan $\arctan x$ funksiyaning darajali qatorga yoyish orqali topilgan.

V. Adabiyotlar

1. https://en.wikipedia.org/wiki/Leibniz_formula_for_%CF%80
2. https://en.wikipedia.org/wiki/Harmonic_series_(mathematics)
3. Usmonov, M.T. & Shokirov.,Sh.H, (2022). Teylor formulasini matematik masalalarni echishdagi ahamiyati. "«Science and Education» Scientific Journal" Scientific Journal, Tom-3, 19-23.
4. Usmonov, M.T. & Shokirov.,Sh.H, (2022). Darajali qatorlarning taqribiy hisoblashlarga tatbiqi. «Science and Education» Scientific Journal, Tom-3, 29-32.

5. Usmonov, M.T. & Shokirov.,Sh.H, (2022). Ishoralari almashinib keluvchi qatorlar. Leybnits alomati. «Science and Education» Scientific Journal, Tom-3, 24-28.

6. Usmonov, M.T. & Shokirov.,Sh.H, (2022). Teylor qatori va uning tadbiqlari. «Science and Education» Scientific Journal, Tom-3, 33-38.

7. Уралов, А. Б., & Бердалиева, С. Х. (2020). МОРФЕМАЛАР ТИЗИМИДА АГГЛЮТИНАЦИЯ ВА ФУЗИЯ МУНОСАБАТИ. Интернаука, (20-4), 81-82.

8. Уралов, А. Б., & Умарова, Б. Х. (2020). ТУРКИЙ ТИЛЛАРДА МОРФЕМАЛАРНИНГ ВУЖУДГА КЕЛИШИ. Интернаука, (20-4), 77-78.

9. Uralov, A. B. (2018). COMPREHENSIVEAND EXTENDED FORMS OF SUFFIXES IN UZBEK LANGUAGE. Theoretical & Applied Science, (12), 127-131.

10. Усмонов, М.Т. (2021). Вычисление центра тяжести плоской ограниченной фигуры с помощью двойного интеграла. «Science and Education» Scientific Journal, Tom-2, 64-71.

11. Усмонов, М.Т. (2021). Биномиальное распределение вероятностей. «Science and Education» Scientific Journal, Tom-2, 81-85.

12. 18. Усмонов,М.Т. (2021). Вычисление определенного интеграла по формуле трапеций и методом Симпсона. «Science and Education» Scientific Journal, Tom-2, 213-225.

13. Усмонов,М.Т. (2021). Метод касательных. «Science and Education» Scientific Journal, Tom-2, 25-34. 20. Усмонов,М.Т. (2021). Вычисление предела функции с помощью ряда. «Science and Education» Scientific Journal, Tom-2, 92-96.

14. Усмонов,М.Т. (2021). Примеры решений произвольных тройных интегралов. Физические приложения тройного интеграла. «Science and Education» Scientific Journal, Tom-2, 39-51. 22. Усмонов,М.Т. (2021). Вычисление двойного интеграла в полярной системе координат. «Science and Education» Scientific Journal, Tom-2, 97-108.

15. Усмонов,М.Т. (2021). Криволинейный интеграл по замкнутому контуру. Формула Грина. Работа векторного поля. «Science and Education» Scientific Journal, Tom-2, 72-80.

16. Усмонов,М.Т. (2021). Правило Крамера. Метод обратной матрицы. «Science and Education» Scientific Journal, Tom-2, 249-255.